AD-756 392

RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS. VOLUME IV. SUBSYSTEMS

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Kaman Aerospace Corporation

Prepared for:

Army Air Mobility Research and Development

November 1972

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USAAMRDL TECHNICAL REPORT 72-63D

RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS

VOLUME IV

SUBSYSTEMS

By

William G. Flannelly

Alex Berman

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EUSTIS DIRECTORATE

U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY

FORT EUSTIS, VIRGINIA

CONTRACT DAAJ02-70-C-0012

KAMAN AEROSPACE CORPORATION

BLOOMFIELD, CONNECTICUT

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This program was conducted under Contract DAAJ02-70-C-0012 with Kaman Aerospace Corporation.

This report contains the theoretical derivation and the presentation of a methodology for system identification of structures. Computer experiments were run to verify this methodology.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. Arthur J. Gustafson, Technology Applications Division.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Kaman Aerospace Corporation Old Windsor Road Bloomfield, Connecticut 06002	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS
VOLUME IV - SUBSYSTEMS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (First name, middle initial, last name)

Nicholas Giansante, William G. Flannelly, Alex Berman

6. REPORT DATE November 1972	7a. TOTAL NO. OF PAGES 51	7b. NO. OF REFS 3

8a. CONTRACT OR GRANT NO. DAAJ02-70-C-0012 b. PROJECT NO. Task 1F162204AA4301 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) USAAMRDL Technical Report 72-63D 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) Kaman Report R-1001-4

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES Volume 4 of a 4-volume report	12. SPONSORING MILITARY ACTIVITY Eustis Directorate U.S. Army Air Mobility R&D Laboratory Fort Eustis, Virginia 23604

13. ABSTRACT

Equations have been derived for determining the dynamic response of the combination of a linear elastic structure and its subsystems. The method is based on measured mobility matrices of the primary structure and the subsystem independently. Mathematical relationships were formulated for the main system and subsystem interface.

The mathematical model established provides for a wide range of cross coupling effects to simulate diverse subsystems. Specifically, the types of subsystems considered were a spring-mass system connected at one point, a rigid inertial mass elastically connected at two points, and a beam elastically attached at three or more points. The spring-mass subsystem attached at one point is illustrative of a simple vibration absorber or a load suspended from the helicopter. The rigid inertial mass subsystem with two-point attachment is typical of a munitions store or fuel storage tank. The beam-type subsystem connected at three or more points is representative of a suspended weapon or possibly an auxiliary engine.

A digital computer program was generated for the IBM Model 360/40 computer using FORTRAN IV language to numerically test the aforementioned theory. Computer experiments were conducted to test the sensitivity of the theory to measurement error in the simulated test data representing the measured mobilities.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
impedance-type test data						
system identification						
orthogonal mode shapes						
generalized mass, stiffness, and damping						
mobility data						
natural frequency						
computer simulations						
computer experiments						
error sensitivity						
simulated test data						
subsystem response						
primary structure						
computer programs						

Task 1F162204AA4301
Contract DAAJ02-70-C-0012
USAAMRDL Technical Report 72-63D
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Volume IV
Subsystems

Final Report

Kaman Report R-1001-4

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Bloomfield, Connecticut

for

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

FOREWORD

The work presented in this report was performed by Kaman Aerospace Corporation under Contract DAAJ02-70-C-0012 (Task 1F162204AA4301) for the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia. The program was implemented under the technical direction of Mr. Joseph H. McGarvey of the Reliability and Maintainability Division* and Mr. Arthur J. Gustafson of the Structures Division.** The report is presented in four volumes, each describing a separate phase of the basic theory of structural dynamic testing using impedance techniques.

Volume I presents the results of an analytical and numerical investigation of the practicality of system identification using fewer measurement points than there are degrees of freedom. The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data. Volume II describes the method of system identification wherein the necessary impedance data are experimentally determined by applying a force excitation at a single point on the structure. Volume III presents a method of determining the free-body dynamic responses from data obtained on a constrained structure. Volume IV describes a method of obtaining the equations for the combination of measured mobility matrices of a helicopter and its subsystems. The response of the combination of a helicopter and its subsystems is determined from data based on the experimental results of the main system and subsystems separately.

**Division name changed to Military Operations Technology Division.

**Division name changed to Technology Applications Division.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF SYMBOLS

Y	Displacement mobility of total system
$\hat{Y}$	Displacement mobility of primary system
Y_B	Free displacement mobility of subsystem
$\hat{Y}_{AA}$	Displacement mobility of primary system, excluding interface points; force excitation on primary system
$\hat{Y}_{AB}$	Displacement mobility of primary system; force excitation at interface points
$\hat{Y}_{BA}$	Displacement mobility of primary system interface points; force excitation at primary system points, excluding interface points
$\hat{Y}_{BB}$	Displacement mobility of interface points; force excitation at interface points
$\hat{Y}^*$	Complex modal mobility of primary system
Z	Impedance of total system
$\hat{Z}_{AA}$	Displacement impedance of primary system, excluding interface points; displacement excitation on primary system
$\hat{Z}_{AB}$	Displacement impedance of primary system, displacement excitation at interface points
$\hat{Z}_{BA}$	Displacement impedance of primary system interface points; displacement excitation at primary system points, excluding interface points
$\hat{Z}_{BB}$	Displacement impedance of interface points; displacement excitation at interface points
$[\Phi_A]$	Modal matrix of primary system, excluding interface points
$[\Phi_B]$	Modal matrix of interface points on primary structure

LIST OF SYMBOLS (Continued)

BRACKETS

[ ], ( )	Matrix
$\lceil \ \rfloor$	Diagonal matrix
{ }	Column or row vector

SUPERSCRIPTS

T	Transpose
-1	Inverse
-T	Transpose of the inverse

INTRODUCTION

The success of a helicopter structural design is highly dependent on the ability to predict and control the dynamic response of the fuselage and appended components. An effective dynamic analysis of complex systems should yield the response of the primary system and its associated subsystems. It is extremely desirable to analytically predict the complete response of a linear elastic structure due to the addition or alteration of particular components.

This report describes a method whereby the dynamic response of the entire system can be determined from knowledge of the response of the main system and the subsystem separately. The formulation is predicated on the theory of structural dynamic testing using impedance techniques. The analysis requires measured mobility matrices for the basic structure alone and free mobility matrices for the attached component. Therefore, once the mobility matrices for the basic system are measured, they can be continually used in conjunction with the measured free mobilities for the various components connected to the main structure.

It is anticipated, in practice, that the mobility matrix for the basic system would be obtained by the method of Volume II of this report and that the free mobilities for the components would be obtained by the method of Volume III.

Specifically, in the present report the primary system was a 20-degree-of-freedom representation of an actual helicopter, and three types of subsystems were considered. The subsystems studied included a spring-mass system connected at a single point, a rigid inertial mass elastically attached at two points, and a beam elastically connected at three or more points. The method employed simulated test data to represent the required experimental mobility data, and measurement errors were introduced to test the sensitivity of the theory to error.

THEORY

Consider a finite degree of freedom simulation of an actual helicopter. The impedance matrix for the system can be expressed in terms of mobilities

$$\left[\begin{array}{c|c} [\hat{Z}_{AA}] & [\hat{Z}_{AB}] \\ \hline [\hat{Z}_{BA}] & [\hat{Z}_{BB}] \end{array}\right] = \left[\begin{array}{c|c} [\hat{Y}_{AA}] & [\hat{Y}_{AB}] \\ \hline [\hat{Y}_{BA}] & [\hat{Y}_{BB}] \end{array}\right]^{-1} \equiv [\hat{Y}]^{-1} \qquad (1)$$

The impedance of a subsystem to be attached to the primary system can also be expressed in terms of mobilities

$$\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [Z_B] \end{array}\right] = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [Y_B]^{-1} \end{array}\right] \equiv [\hat{Y}_B]^{-1} \qquad (2)$$

The mobility of the combined system is defined as the inverse of the impedance

$$[Y] \equiv \left(\left[\begin{array}{c|c} [\hat{Z}_{AA}] & [\hat{Z}_{AB}] \\ \hline [\hat{Z}_{BA}] & [\hat{Z}_{BB}] \end{array}\right] + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [Y_B]^{-1} \end{array}\right] \right)^{-1} \qquad (3)$$

The product of the impedance matrix and the mobility matrix is the unit matrix

$$[Z]\,[Y] = \left([\hat{Y}]^{-1} + [\bar{Y}_B]^{-1}\right) [Y] = [I] \qquad (4)$$

Multiplying both sides of Equation (4) by $[\hat{Y}]$ and solving for $[Y]$ yields

$$[Y] = \left([I] + [\hat{Y}]\,[\bar{Y}_B]^{-1}\right)^{-1} [\hat{Y}] \qquad (5)$$

Substituting the actual matrices into the matrix Equation (5),

$$
\begin{bmatrix} [Y_{AA}] & [Y_{AB}] \\ [Y_{BA}] & [Y_{BB}] \end{bmatrix} = \left(\begin{bmatrix} [I] & 0 \\ 0 & [I] \end{bmatrix} + \begin{bmatrix} [\hat{Y}_{AA}] & [\hat{Y}_{AB}] \\ [\hat{Y}_{BA}] & [\hat{Y}_{BB}] \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & [Y_B]^{-1} \end{bmatrix} \right)^{-1} \begin{bmatrix} [\hat{Y}_{AA}] & [\hat{Y}_{AB}] \\ [\hat{Y}_{BA}] & [\hat{Y}_{BB}] \end{bmatrix} \tag{6}
$$

Performing the indicated operations within the matrix inverse results in

$$
\begin{bmatrix} [Y_{AA}] & [Y_{AB}] \\ [Y_{BA}] & [Y_{BB}] \end{bmatrix} = \begin{bmatrix} [I] & [\hat{Y}_{AB}][\hat{Y}_B]^{-1} \\ 0 & [I]+[\hat{Y}_{BB}][Y_B]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} [\hat{Y}_{AA}] & [\hat{Y}_{AB}] \\ [\hat{Y}_{BA}] & [\hat{Y}_{BB}] \end{bmatrix} \equiv [X][\hat{Y}] \tag{7}
$$

To facilitate solution of the matrix Equation (7), the inverse of the matrix on the right side of the equation must be evaluated. Let

$$
\begin{bmatrix} [X_{11}] & [X_{12}] \\ [X_{21}] & [X_{22}] \end{bmatrix} \begin{bmatrix} [I] & [Y_{AB}][Y_B]^{-1} \\ 0 & [I]+[\dot{Y}_{BB}][Y_B]^{-1} \end{bmatrix} = \begin{bmatrix} [I] & 0 \\ 0 & [I] \end{bmatrix} \tag{8}
$$

Therefore

$$
[X_{11}] = [I], \quad [\hat{Y}_{AB}[Y_B]^{-1} + [X_{12}]([I] + \hat{Y}_{BB}[Y_B]^{-1}) = 0
$$

$$
X_{21} = 0, \qquad [X_{22}]([I] + [\hat{Y}_{BB}][Y_B]^{-1}) = [I] \tag{9}
$$

Solving for the elements of the X matrix yields

$$
\begin{bmatrix} [X_{11}] & [X_{12}] \\ [X_{21}] & [X_{22}] \end{bmatrix} = \begin{bmatrix} [I] & -[\hat{Y}_{AB}][Y_B]^{-1}\left([I] + [\hat{Y}_{BB}]^{-1}\right)^{-1} \\ 0 & \left([I] + [\hat{Y}_{BB}][Y_B]^{-1}\right)^{-1} \end{bmatrix} \tag{10}
$$

Substituting the X matrix as expressed in Equation (10) into Equation (7) and expanding gives

$$\left[\begin{array}{c|c} [Y_{AA}] & [Y_{AB}] \\ \hline [Y_{BA}] & [Y_{BB}] \end{array}\right] = \left[\begin{array}{c|c} [\hat{Y}_{AA}] - [\hat{Y}_{AB}]\left([Y_B]+[\hat{Y}_{BB}]\right)^{-1}[\hat{Y}_{BA}] & [\hat{Y}_{AB}] - [\hat{Y}_{AB}]\left([Y_B]+[\hat{Y}_{BB}]\right)^{-1}[\hat{Y}_{BB}] \\ \hline \left([I]+[\hat{Y}_{BB}][Y_B]^{-1}\right)^{-1}[\hat{Y}_{BA}] & \left([I]+[\hat{Y}_{BB}][Y_B]^{-1}\right)^{-1}[\hat{Y}_{BB}] \end{array}\right] \tag{11}$$

Equation (11) can be simplified to

$$\left[\begin{array}{c|c} [Y_{AA}] & [Y_{AB}] \\ \hline [Y_{BA}] & [Y_{BB}] \end{array}\right] = \left[\begin{array}{c|c} [\hat{Y}_{AA}] & [\hat{Y}_{BA}] \\ \hline 0 & 0 \end{array}\right] - \left[\begin{array}{c|c} [\hat{Y}_{AB}] & [\hat{Y}_{AB}] \\ \hline [-Y_B] & [-Y_B] \end{array}\right] \left[\begin{array}{c|c} \left([Y_B]+[\hat{Y}_{BB}]\right)^{-1}[\hat{Y}_{BA}] & 0 \\ \hline 0 & \left([Y_B]+[\hat{Y}_{BB}]\right)^{-1}[\hat{Y}_{BB}] \end{array}\right] \tag{12}$$

Equation (12) yields the response of the complete system for force excitation at each position on the main system including the interface points. Since the force excitation on a helicopter is usually applied at one particular point, Equation (12) can be reduced, yielding the complete structural response for forcing at a single point on the structure. Thus, Equation (12) reduces to a column of mobilities for force excitation applied at position j of the structure

$$\left\{\begin{array}{c} Y_{AA} \\ \hline Y_{BA} \end{array}\right\}_j = \left\{\begin{array}{c} Y_{AA} \\ \hline 0 \end{array}\right\}_j - \left[\begin{array}{c} [\hat{Y}_{AB}] \\ [-Y_B] \end{array}\right] \left[\left([Y_B]+[Y_{BB}]\right)^{-1}\right] \left\{\hat{Y}_{BA}\right\}_j \tag{13}$$

where $[Y_{AA}]$ represents the dynamic response of each position on the primary system excluding the interface points and $[Y_{BA}]$ describes the response of the attachment points.

It is possible to obtain the dynamic response for the complete system utilizing the modal matrix of the points of interest on the primary structure exclusive of attachment points, the modal matrix of the interface points and the complex modal mobility of the primary system and the free mobility of the appended subsystem. These parameters can be obtained employing the techniques described in References 1 and 2. Following the method of Reference 2, the mobility of the main system exclusive of the subsystem connection points is given by

$$[Y_{AB}] = [\hat{\Phi}_A]\lceil \hat{Y}^* \rfloor[\hat{\Phi}_B]^T \tag{14}$$

where the number of rows of the matrix $\left[\hat{\Phi}_A\right]$ corresponds to the number of points of interest on the main system, exclusive of the subsystem attachment points, and the number of columns corresponds to the number of modes and is equal to the total number of points of interest on the primary system including the interface points. Matrix $\left[\hat{\Phi}_B\right]$ has the same number of columns as the number of connection points between the two systems and the identical number of columns as matrix $\left[\hat{\Phi}_A\right]$. The mobility of the subsystem attachment point is

$$[\hat{Y}_{BB}] = [\hat{\Phi}_B]\lceil \hat{Y}^* \rfloor[\Phi_B]^T \tag{15}$$

If Equations (14) and (15) are substituted in Equation (13), the result is

$$\begin{Bmatrix} Y_{AA} \\ \hline Y_{BA} \end{Bmatrix}_j = \begin{Bmatrix} \hat{Y}_{AA} \\ \hline 0 \end{Bmatrix}_j - \begin{bmatrix} [\hat{\Phi}_A]\lceil \hat{Y}^* \rfloor[\hat{\Phi}_B]^T \\ \hline - [Y_B] \end{bmatrix} \Bigg[[Y_B] + [\Phi_B]\lceil \hat{Y}^* \rfloor[\hat{\Phi}_B]^T \Bigg]^{-1} \begin{Bmatrix} \hat{Y}_{BA} \end{Bmatrix}_j \tag{16}$$

ERROR ANALYSIS

Measurements of the complex mobilities will be subject to experimental errors of various types, including errors in equipment calibration, errors resulting from equipment incompatibility, errors due to extraneous signals and errors due to random noise.

Generally, all errors can be classified as either random or bias. The random errors are equally likely to be positive or negative, whereas the bias errors are systematic and in one direction only. In the present study, both types of measurement error have been included. The simulated experimental data were polluted with measurement errors of ±5 percent random and 5 percent bias or both the real and imaginary components of displacement mobility.

As indicated in Reference 3, there is no definitive probability distribution for errors of each type in impedance testing practice. In the present analysis, a random number generator was utilized with a resultant uniform distribution of random error. The rectangular distribution of accidental type error between the selected limits is very conservative compared to the usual definition of the limits at three standard deviations from the mean of a normal distribution.

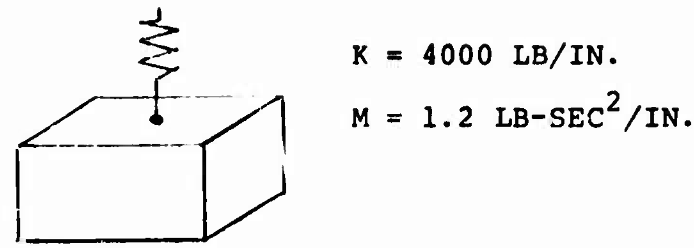
COMPUTER SIMULATION RESULTS

A computer study to determine the response of the combination of a helicopter and its subsystems based on the simulated test results of the individual system and subsystems can be extremely useful in the development cycle of a helicopter. In the present analysis, a mathematical model was established to provide for a wide range of cross-coupling effects to simulate diverse subsystems. The helicopter, or main system, was represented by a 20-degree-of-freedom mathematical model. Table I presents a lumped mass description of the aforementioned specimen which was used to generate the simulated experimental data. Three types of subsystems were incorporated in the study, represented as a spring-mass system elastically connected at one point, a rigid inertial mass elastically connected at two points and a beam elastically connected at three or more points. Figure 1 shows the aforementioned subsystems.

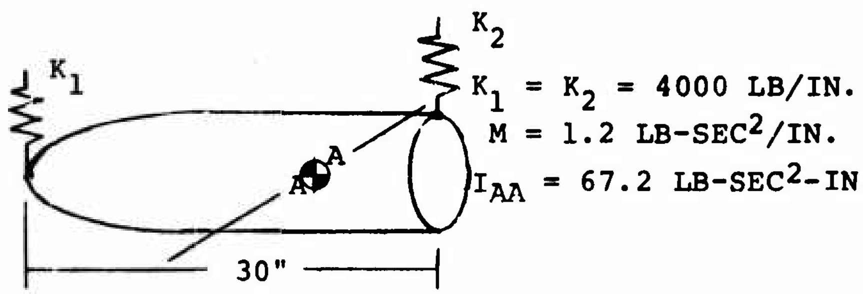
Figures 2 and 3 present the real and imaginary displacement mobility frequency response for the main system-subsystem interface point for a spring-mass subsystem. The force excitation was applied at Station 6, the hub station, and the response was measured at Station 3, the general area of the pilot seat, and coincident with the subsystem attachment point. Data are presented for conditions of zero experimental error and for simulated experimental displacement mobility data recorded with a random error of ± 5 percent and a bias error of +5 percent. For the cases involving error, the random displacement mobility error was computed using a uniformly distributed probability density function. This error was applied to both the real and imaginary components of the main and subsystem displacement mobility data. As can be observed from Figures 2 and 3, the method is extremely insensitive to the measurement error as applied herein. Figures 4 and 5 show the same type data as given in the previous figures except that the subsystem investigated was a rigid inertial mass elastically attached to the main structure at two points. The interface points in this situation are located at Stations 1 and 2 of the main system. Again, the frequency response of the displacement mobility, both real and imaginary, is effectively invariant with error for the error level incorporated. Figures 6 and 7 present the same type results for the beam subsystem elastically connected at three points. The attachment points in this instance are at Stations 1, 2, and 3 of the main system. The data substantiate the previous observations of the relative insensitivity of the method to error.

TABLE I. 20-POINT MODEL DESCRIPTION

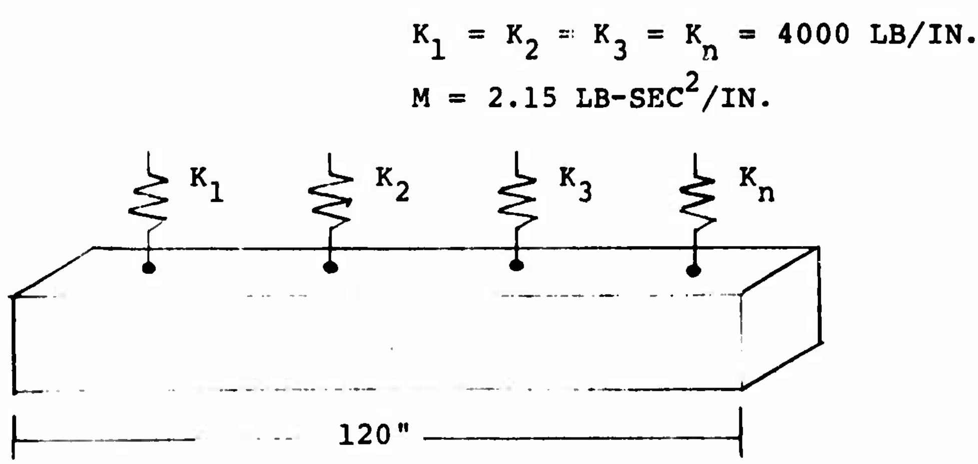
Sta No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sta (In.)	0		60		120		160		200		240		280		320		370		430	
		30		100		140		180		220		260		300		340		400		460
Mass (Lb-Sec^2/In.)	.029		3.67		2.18		2.385		2.08		.910		.170			.070		.095	.210	
		1.05		3.71		2.18		2.59		1.56		.260		.085		.060		.120		.150
EI (Lb-In.^{2} x 10^{10})	.35		.35		1.95		4.37		5.80		4.425		3.07		2.05		.975		.55	
		.35		1.20		3.00		5.70		5.60		3.6		2.60		1.60		.65		.50
Springs to Ground (Lb/In.)						10,000												10,000		



SPRING-MASS SUBSYSTEM



RIGID INERTIAL MASS SUBSYSTEM



BEAM SUBSYSTEM

Figure 1. Subsystem Representation.

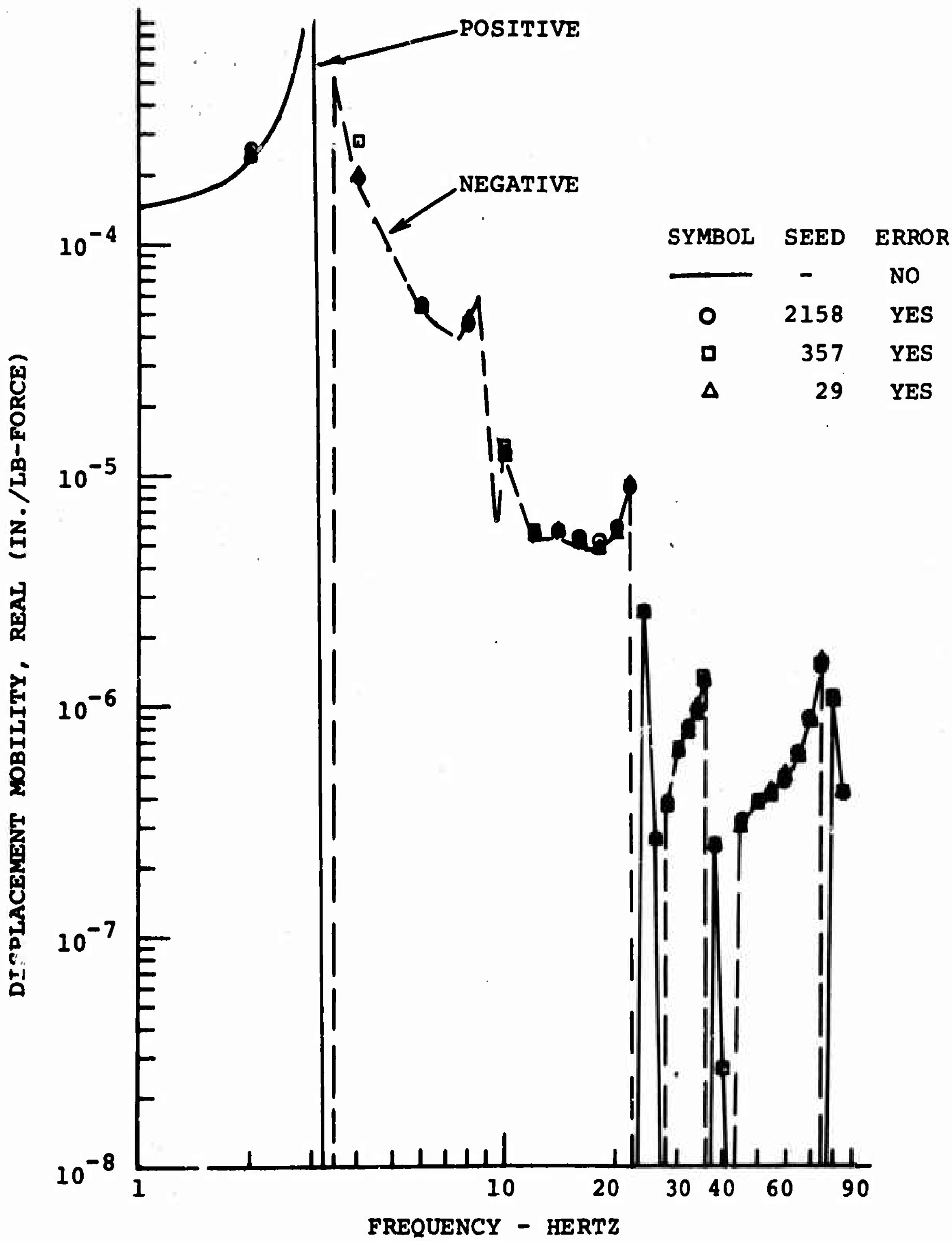


Figure 2. Real Displacement Mobility Frequency Response; Combination of Main System and Spring-Mass Subsystem.

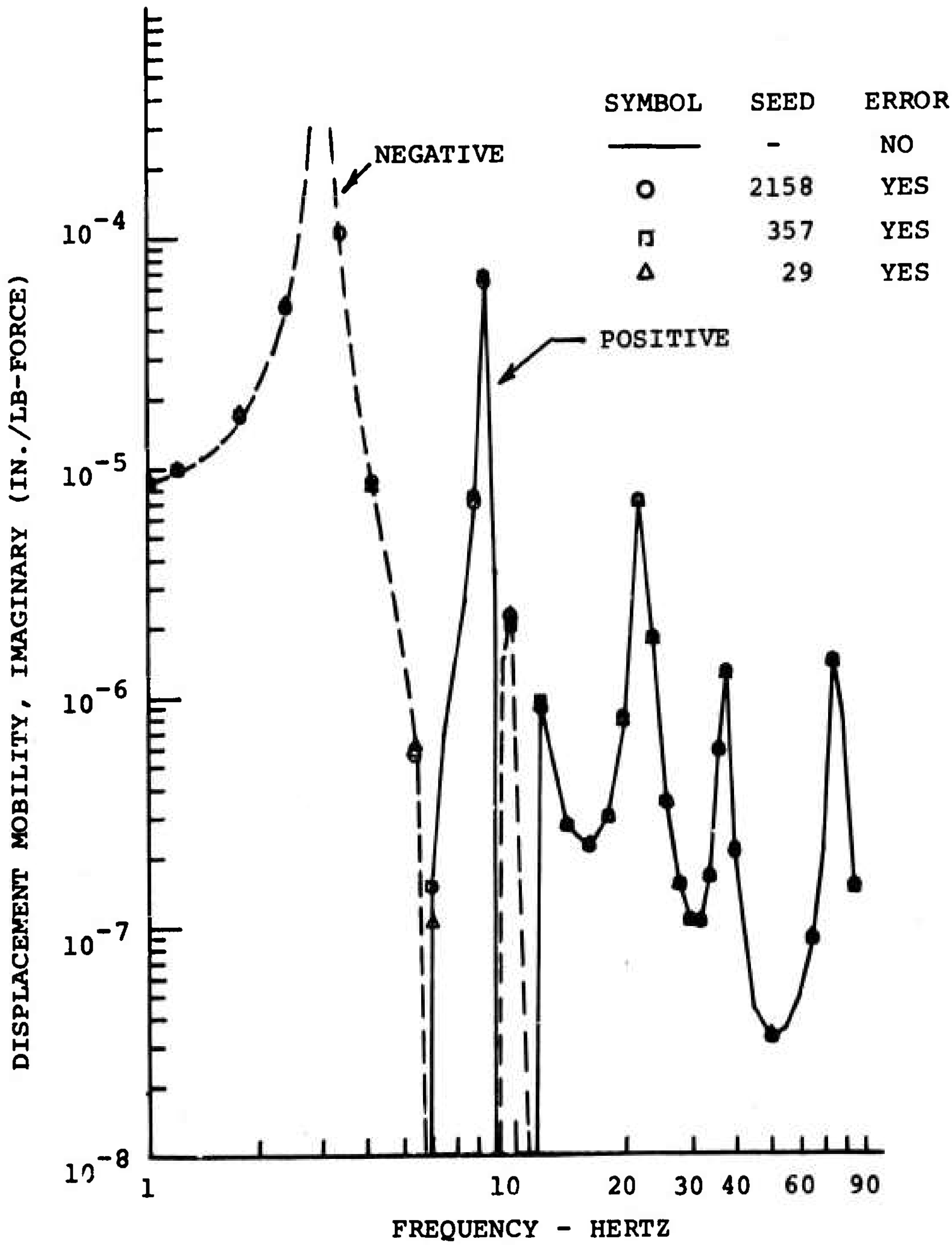


Figure 3. Imaginary Displacement Mobility Frequency Response; Combination of Main System and Spring-Mass Subsystem.

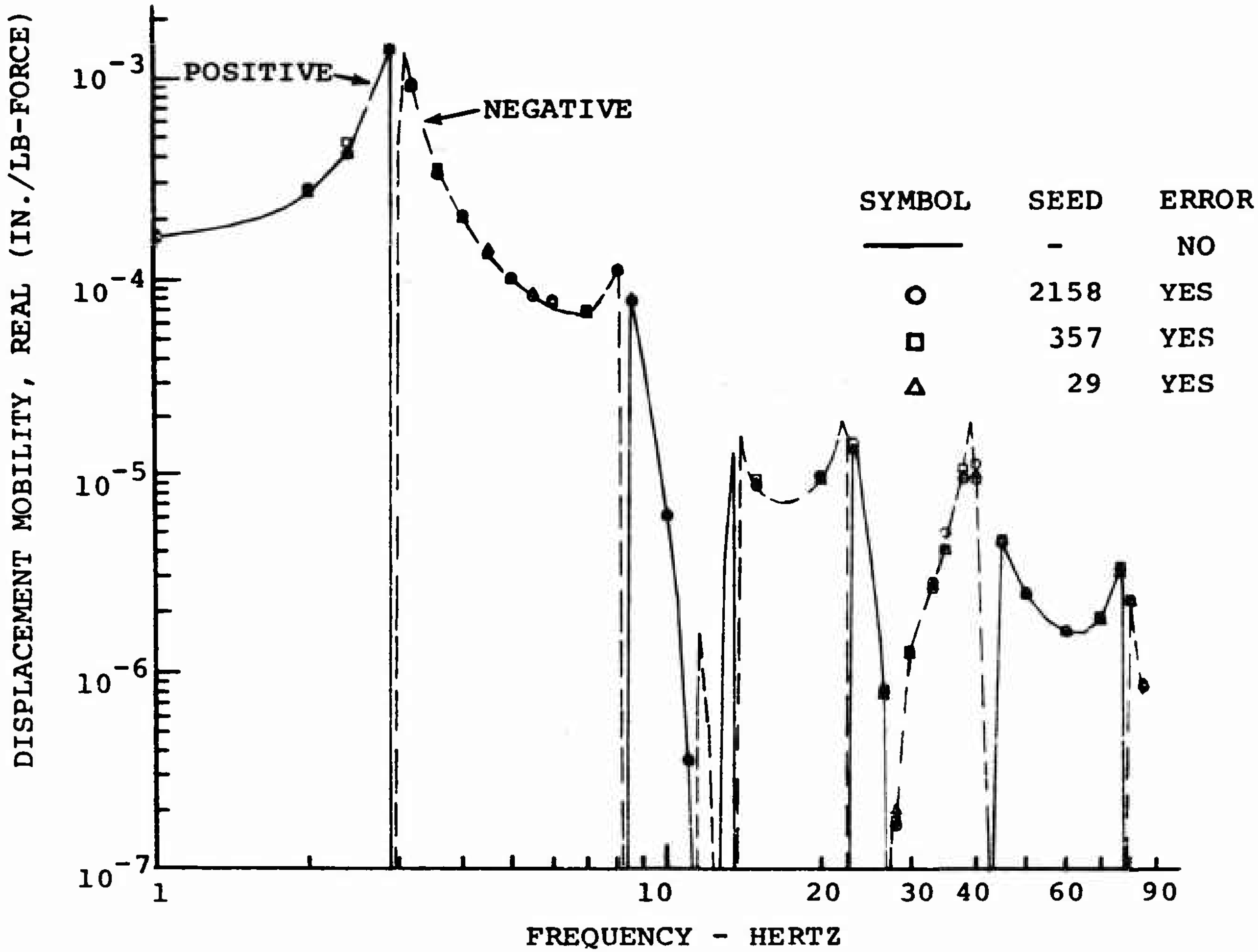


Figure 4. Real Displacement Mobility Frequency Response; Combination of Main System and Rigid Inertial Mass Subsystem.

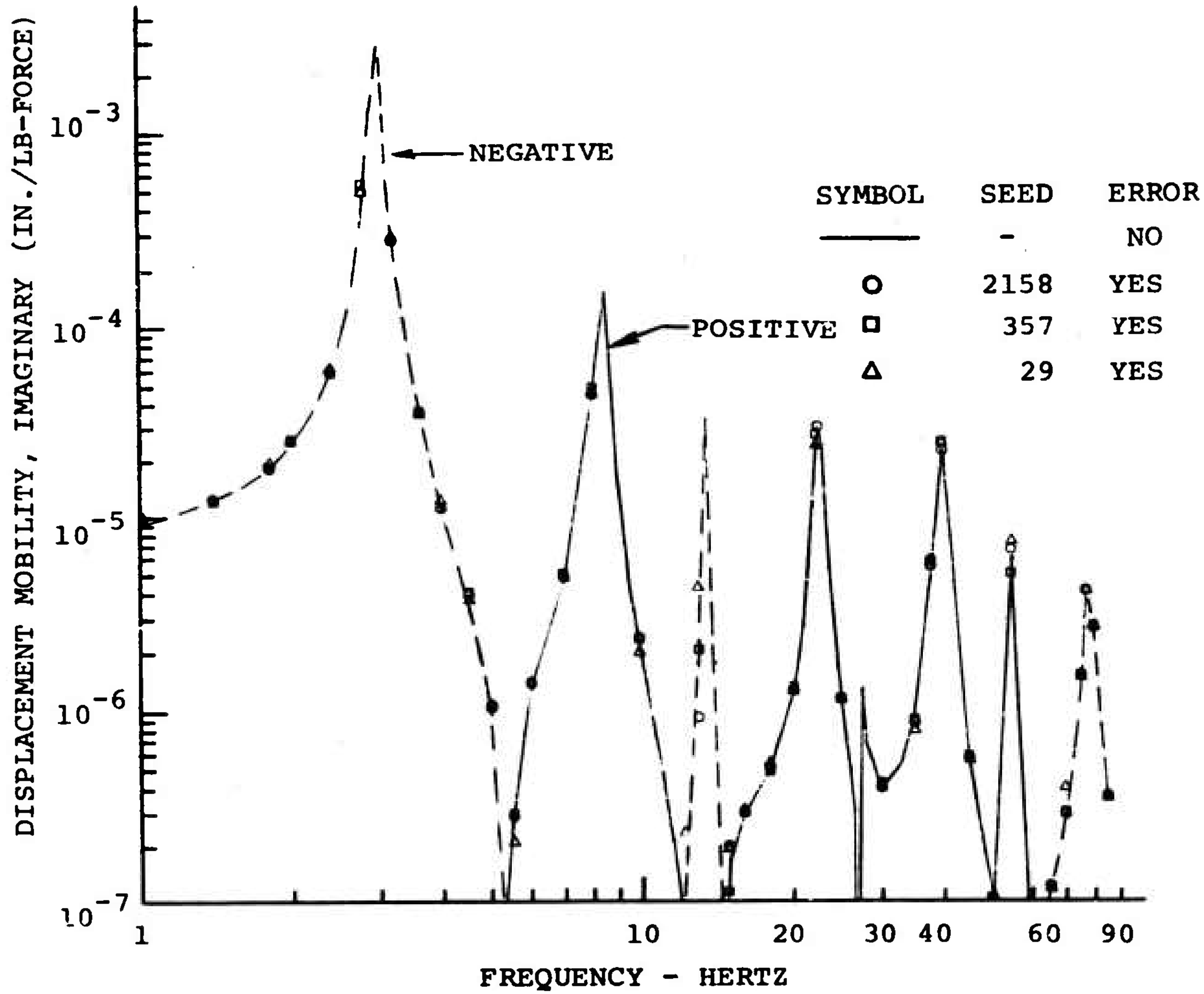


Figure 5. Imaginary Displacement Mobility Frequency Response; Combination of Main System and Rigid Inertial Mass Subsystem.

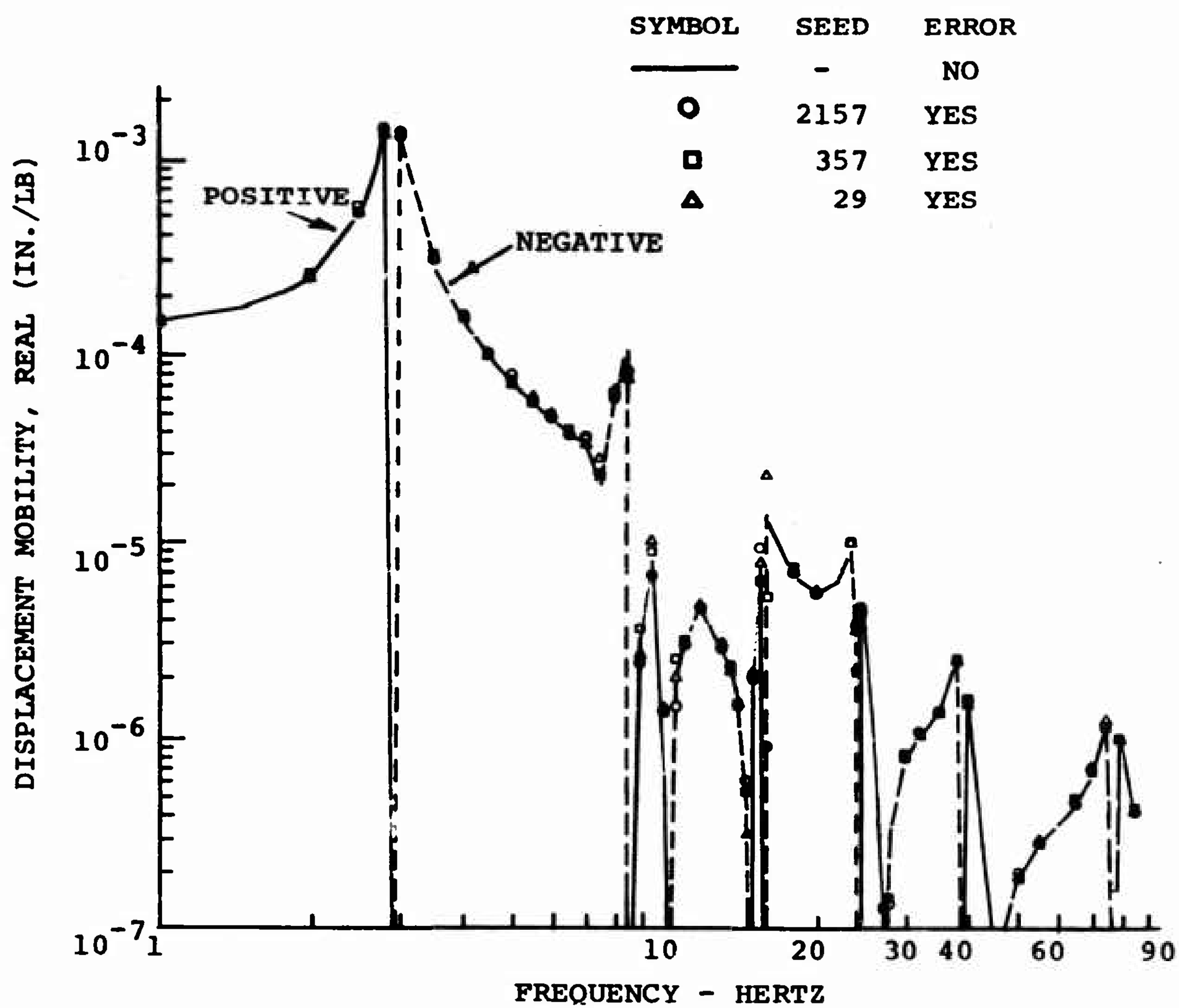


Figure 6. Real Displacement Mobility Frequency Response; Combination of Main System and Beam Subsystem.

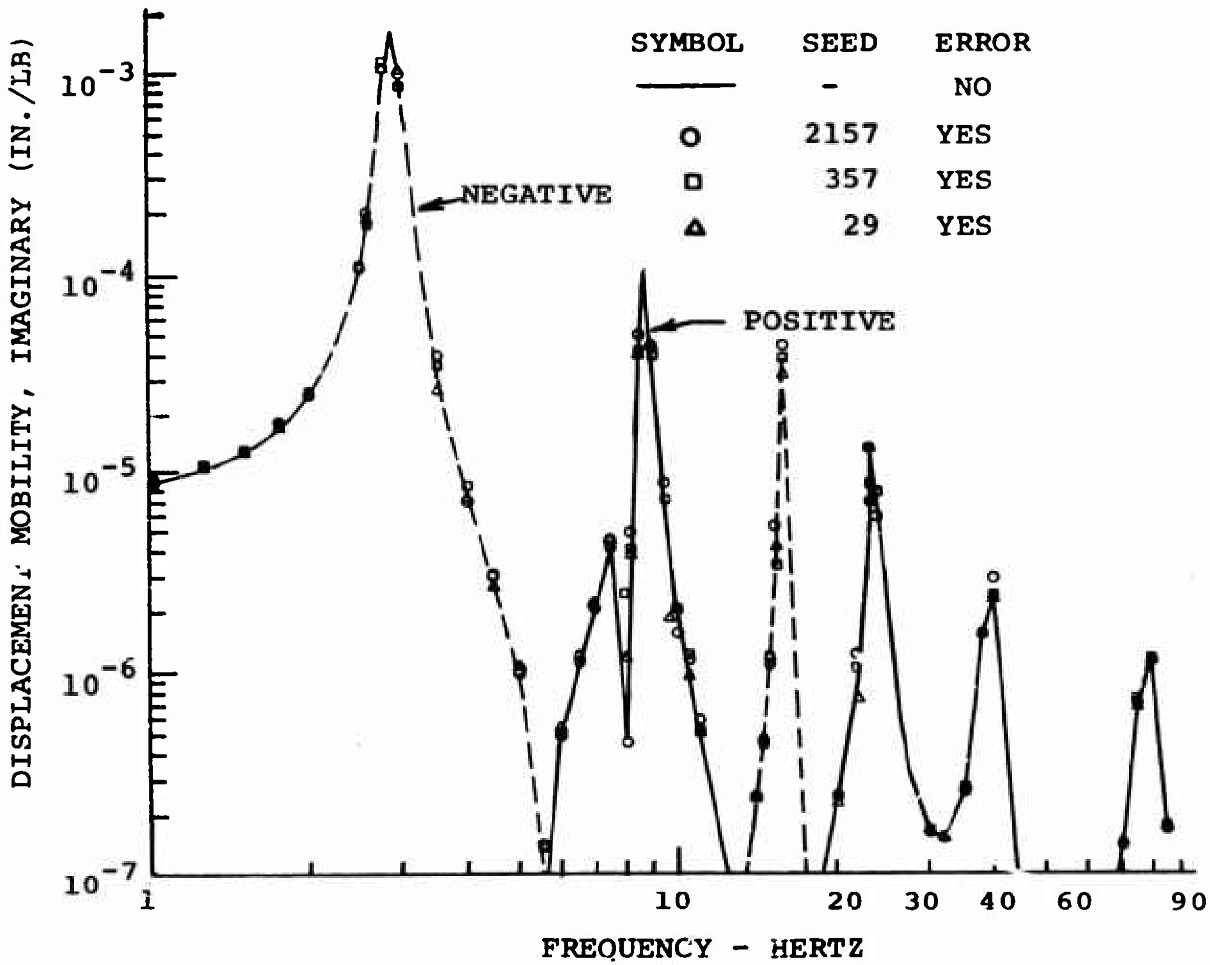


Figure 7. Imaginary Displacement Mobility Frequency Response; Combination of Main System and Beam Subsystem.

The effect of a spring-mass subsystem on the response of the 20-degree-of-freedom mathematical representation of the helicopter is shown in Figure 8. A sinusoidal force excitation was applied at the hub station, and the real displacement mobility is given for the basic system alone and for the main system with subsystem attached at Station 3, the pilot seat location. The forcing frequency was 9.2 Hertz, which is the calculated natural frequency of the spring-mass subsystem alone. As would be expected, the response of the total system is characterized by a nodal point at the interface station. The effect of error is also indicated on the figure. The dispersion of the results using the various random error seeds is within an acceptable level. Figure 9 presents similar data as Figure 8 except that the forcing frequency is 20 Hertz. The disturbing frequency is separated enough from the subsystem resonant frequency that there is essentially no difference in the system response with or without subsystem attached. Figures 10 and 11 illustrate the influence of the rigid inertial mass subsystem on the response of the combination of the basic helicopter representation and the appended subsystem. The forcing frequencies used represent the approximate natural frequencies of the subsystem alone. The effect of error can be observed from the figures, and the insensitivity of the analysis to measurement error is again visible. The response of the configuration, consisting of the main system and a beam elastically connected at three points on the main structure, is represented in Figure 12 for an excitation frequency of 10 Hertz and in Figure 13 for a forcing frequency of 30 Hertz. The figures also yield the effect of measurement error on the system displacement mobility response.

The results shown represent a small number of the computer simulation experiments actually conducted. For each of the subsystem configurations considered, the location of the attachment points along the main system could be varied as long as compatibility between the systems was maintained. The flexibility of the method presented in this report in determining the response of the combination of a helicopter and its subsystems based on test results of the individual system and subsystems provides a means of modifying subsystem characteristics, interface locations and connection effects. A wide range of cross-coupling effects to simulate diverse subsystems can be analyzed based on the measured mobilities of the individual subsystems, since the mobility data of the main system, once measured, remain constant.

A digital computer program listing in FORTRAN IV language and a description of the program input cards are given in the appendix.

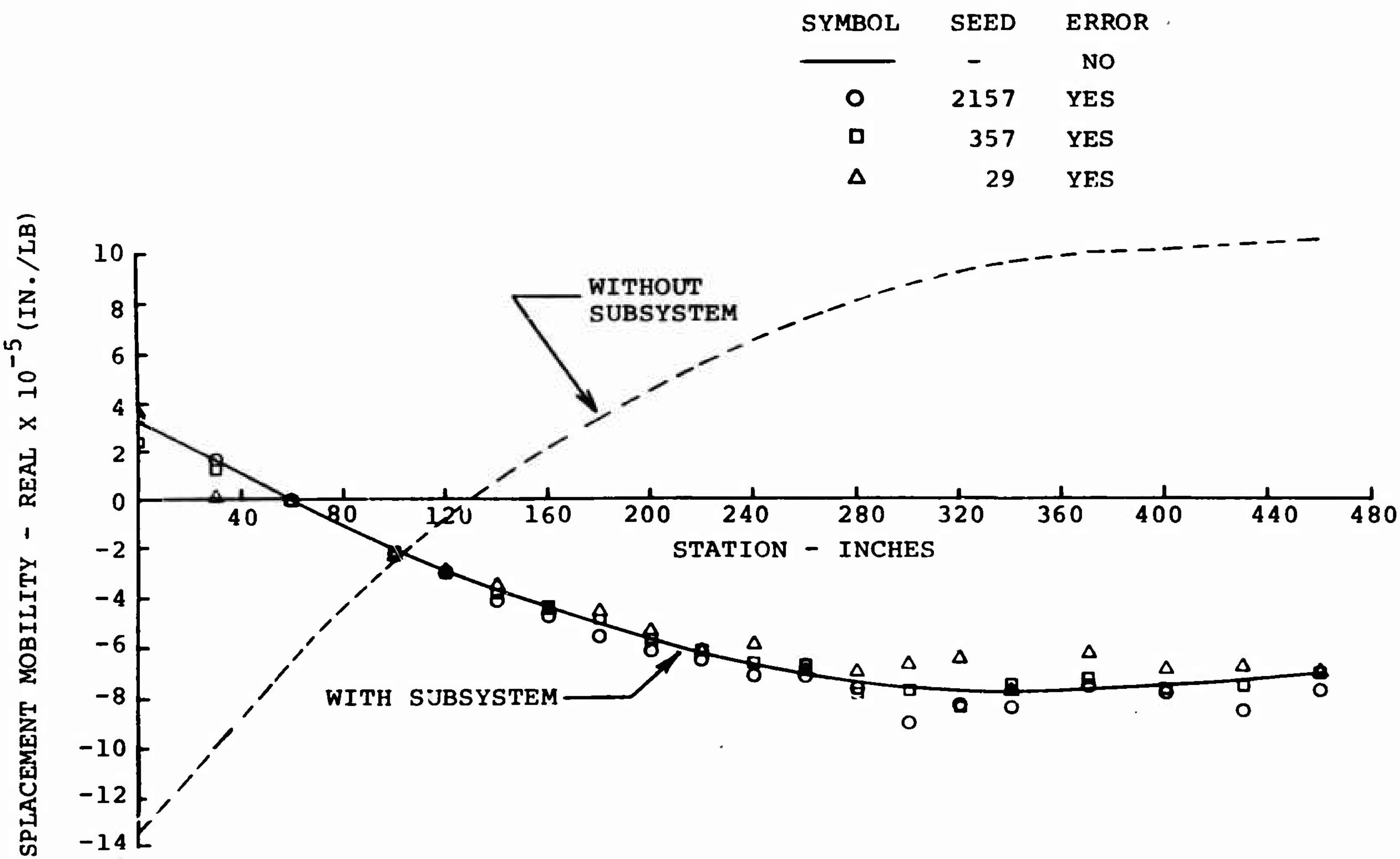


Figure 8. 20-Degree-of-Free'om Model; Real Displacement Mobility; Effect of Spring-Mass Subsystem. Frequency = 9.2 Hertz.

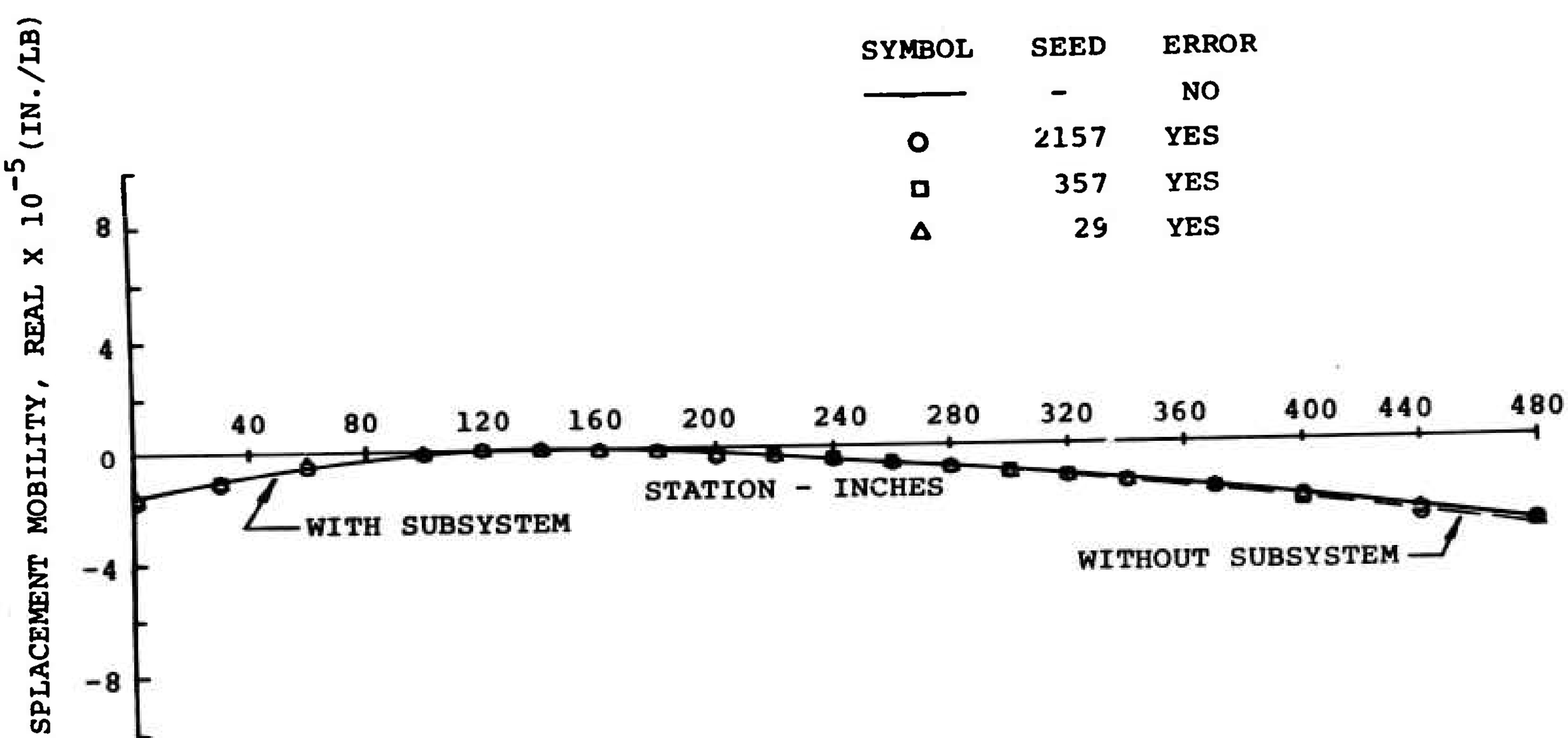


Figure 9. 20-Degree-of-Freedom Model; Real Displacement Mobility; Effect of Spring-Mass Subsystem. Frequency = 20 Hertz.

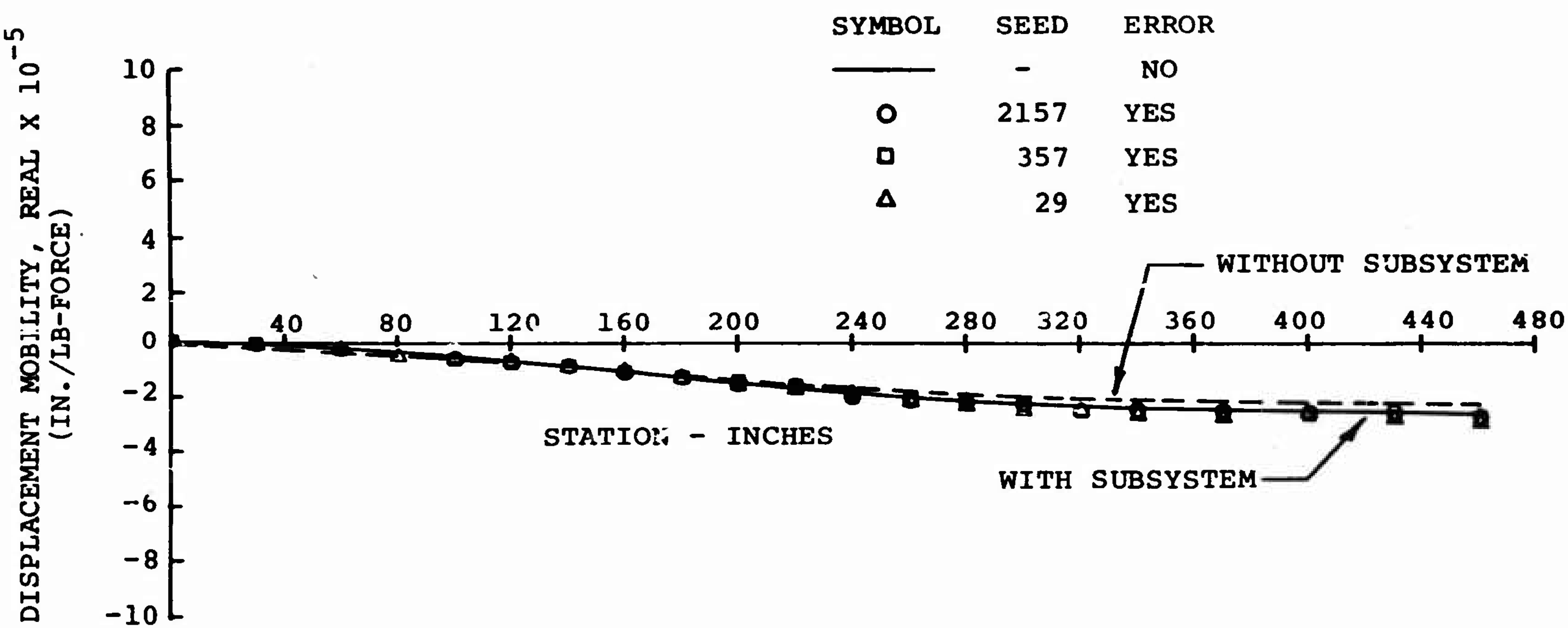


Figure 10. 20-Degree-of-Freedom Model; Real Displacement Mobility; Effect of Rigid Inertial Mass Subsystem. Frequency = 12.1 Hertz.

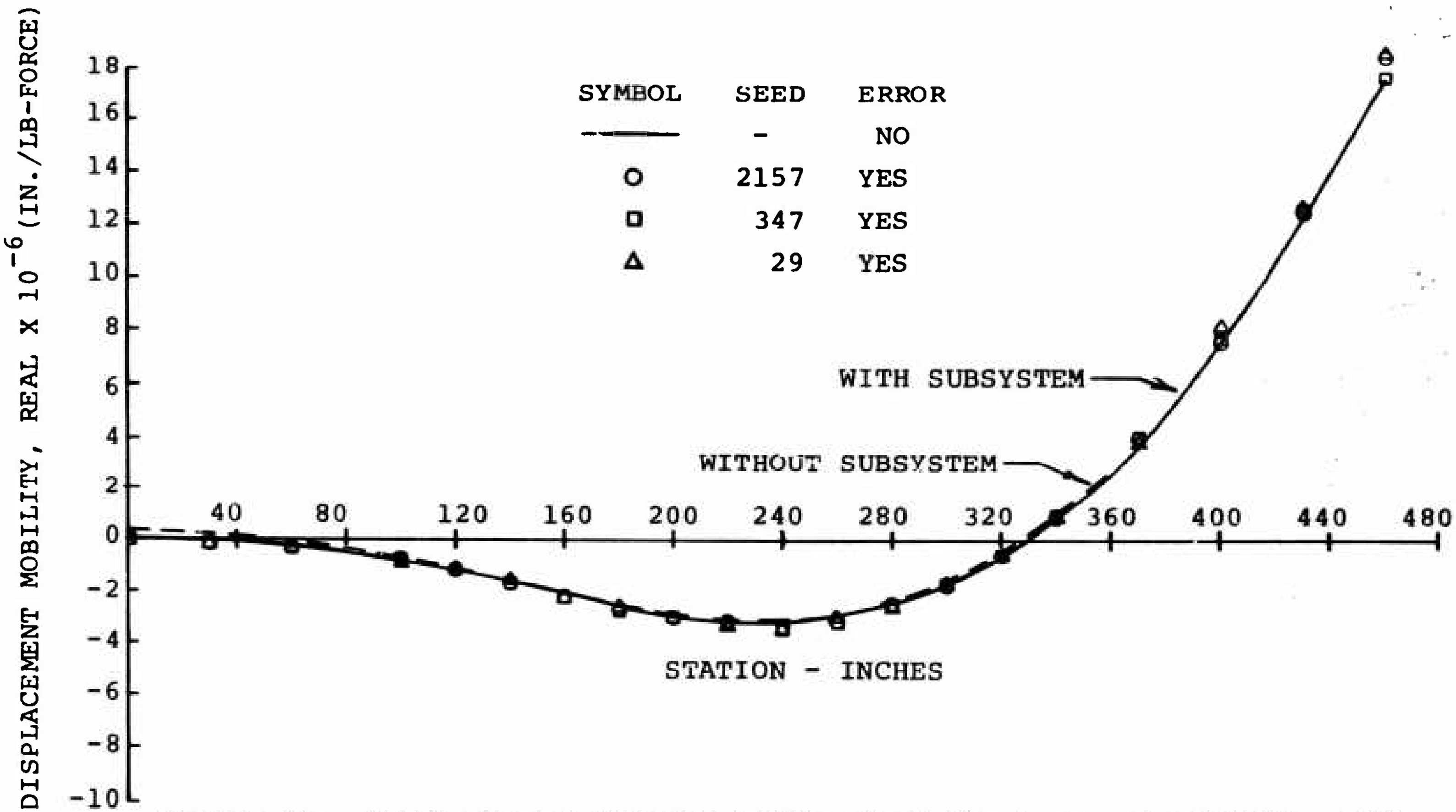


Figure 11. 20-Degree-of-Freedom Model; Real Displacement Mobility; With and Without Rigid Inertial Mass Subsystem. Frequency = 28 Hertz.

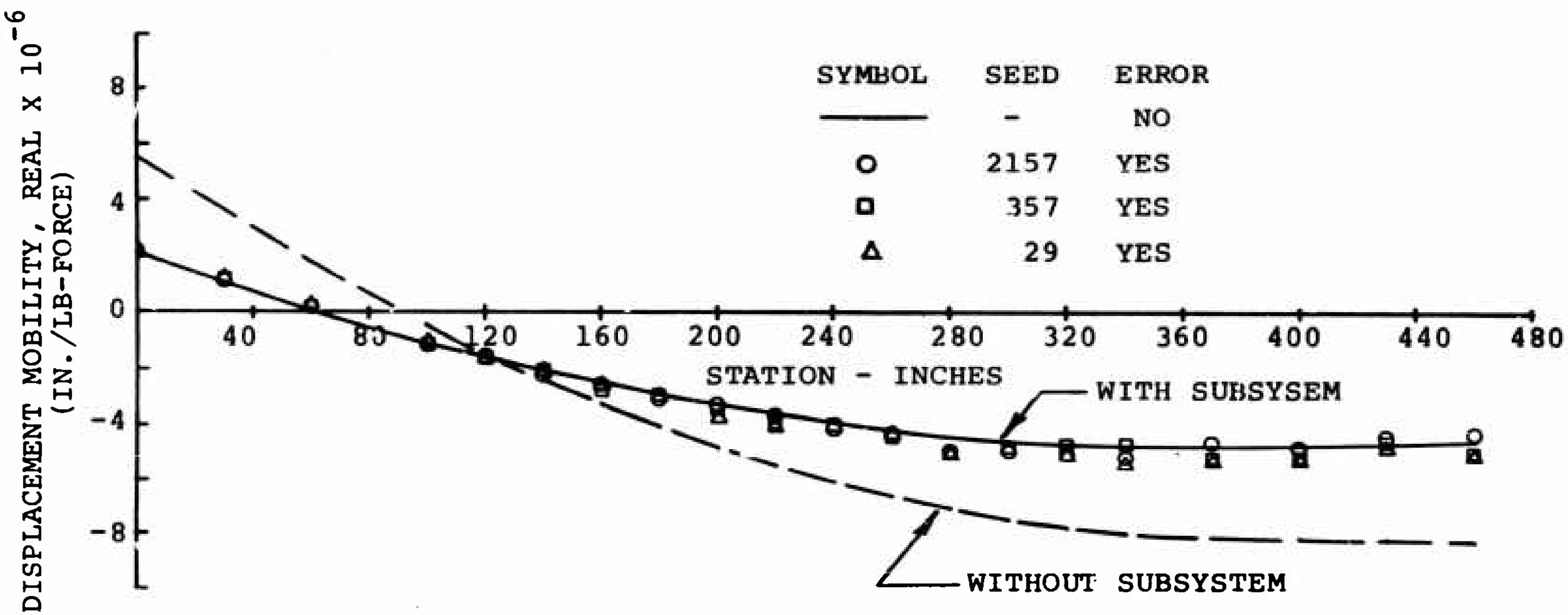


Figure 12. 20-Degree-of-Freedom Model; Real Displacement Mobility; Effect of Beam Subsystem. Frequency = 10 Hertz.

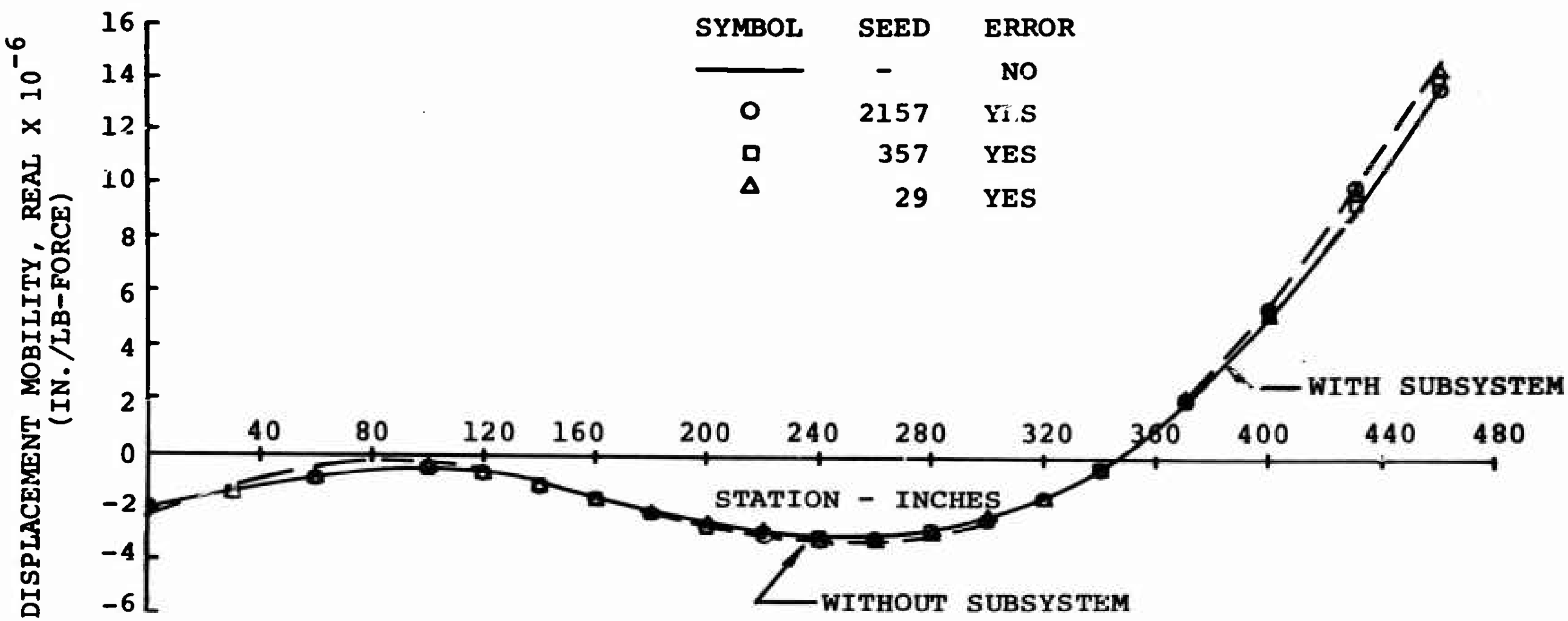


Figure 13. 20-Degree-of-Freedom Model; Real Displacement Mobility; Effect of Beam Subsystem. Frequency = 30 Hertz.

CONCLUSIONS

1. The response of the combination of a helicopter and its subsystems can be determined based on the test results of the individual system and subsystems.

2. The method is insensitive to measurement error using simulated test data subjected to errors that are within the state of the measurement art.

3. The method can be used to study a wide range of cross-coupling effects to simulate diverse subsystems.

4. At a specific excitation frequency, once the mobility data for the main system are measured, they remain constant; thus, only the measured mobilities of the individual subsystems need be considered.

5. The method provides an expedient means of modifying the subsystems appended to a helicopter at a development stage of the system.

6. The method is amenable to experimental implementation and is numerically sound.

LITERATURE CITED

1. Flannelly, W.G., Berman, A., Giansante, N., RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS - PHASE I REPORT, Kaman Aerospace Corporation; USAAMRDL Technical Report 72-63A, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia November 1972.

2. Giansante, N., Flannelly, W.G., Berman, A., RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS - PHASE II Report 72-63B, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia November 1972.

3. Flannelly, W.G., Berman, A., Barnsby, R.M., THEORY OF STRUCTURAL DYNAMIC TESTING USING IMPEDANCE TECHNIQUES - VOLUME 1 - THEORETICAL DEVELOPMENT, Kaman Aerospace Corporation;USAAVLABS Technical Report 70-6A, U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia, June 1970, AD 874509.

APPENDIX
COMPUTER PROGRAM DESCRIPTION

A digital program was prepared for determining the response of the combination of a helicopter and its subsystems based on the test results of the individual system and subsystems. The program was written for the IBM 360/40 disk operating system using FORTRAN IV language. A flow chart depicting the program logic is shown in Figure 14. A description of the input cards and a program source listing are presented in this appendix.

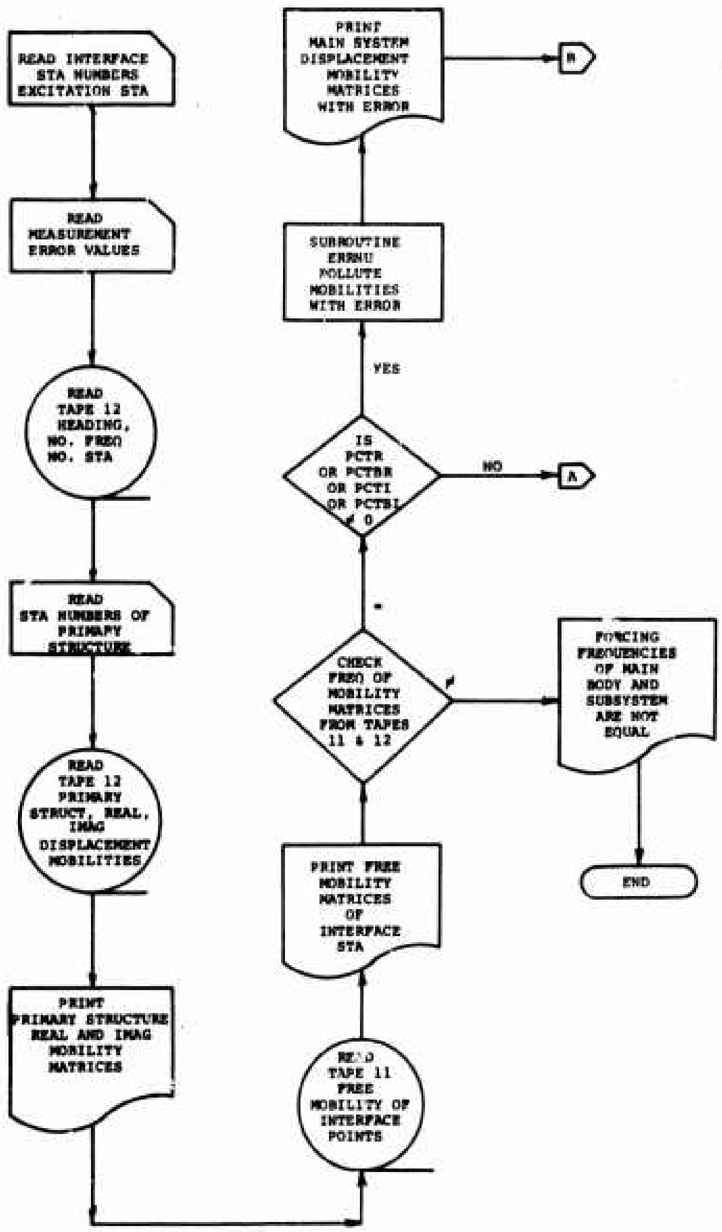


Figure 14. Computer Program Flow Chart.

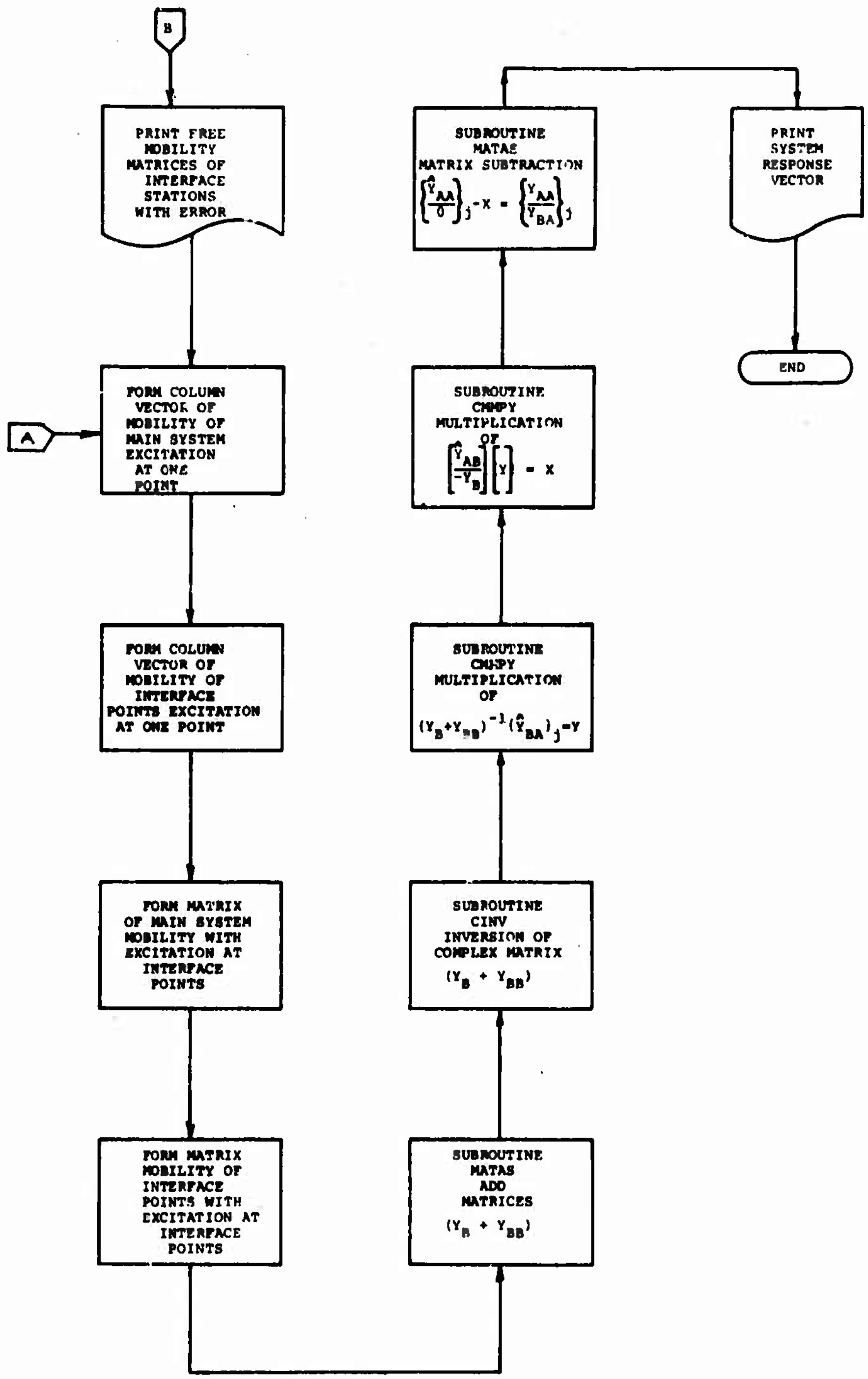


Figure 14 - Concluded.

DESCRIPTION OF INPUT CARDS

Note: All integer variables must be right justified with no decimal point.

Tape, Card Reader and Printer Assignments

1 Card Reader

3 Printer

11 Contains free displacement mobility matrices for subsystem connection points.

12 Contains displacement mobility matrices for primary system.

All input data must be in the following units:

Mass - $lb\text{-}sec^2/in.$

Stiffness - lb/in.

Frequencies - Hz

PROGRAM COMSYSA
COMPONENT SYNTHESIS

Card(s)	1	Columns	1-10	M	Number of interface points (Number of attachment points between subsystem and primary system (FORMAT I10).
				MS	Interface points, station numbers. 10 columns per value (Maximum 10 values). (FORMAT 8I10).
				NCØL	Primary system station number at which forcing function is applied (FORMAT I10).
Card	2		1-10	PCTR	Random error applied to real mobility. Uniform between - and + PCTR* ELEMENT displacement.
			11-20	PCTBR	Bias error applied to real mobility. PCTBR* ELEMENT displacement.
			21-30	PCTI	Same as PCTR except applied to imaginary displacement mobility.
			31-40	PCTBI	Same as PCTBR except applied to imaginary displacement mobility.
			41-50	IZ	Random error seed, used in generation of error.
Card(s)	3		1-10	KEEP	Stations to be used in primary system. ω columns per value, 8 values per card (FORMAT 8I10).

```
C     COMPONENT SYNTHESIS
C
      DIMENSION MS(20),HT(7),HEAD(20),HZ(100),YR(20,21),YI(20,21),
     AYABR(20,21),YABI(20,21),YCCR(20,21),YCCI(20,21),YAAR(20,21),
     BYAAI(20,21),YBR(20,21),YBI(20,21),HF(100),YCR(20,21),YCI(20,21),
     CYSR(20),YSI(20),KEEP(20)
      READ (1,100) M,(MS(I),I=1,M),NCOL
  100 FORMAT (8I10 )
      READ (1,300) PCTR,PCTBR,PCTI,PCTBI,IZ
      WRITE (3,110) PCTR,PCTBR,PCTI,PCTBI,IZ
  110 FORMAT ('1',T10,'MAX RAND ERROR ON REAL      ='F6.3,10X,'BIAS ERROR
     A ON REAL      ='F6.3,' OF ELEMENTS'/T10,'                ON IMAGINAR
     BY='F6.3,21X'ON IMAGINARY='F6.3,15X,'SEED='I5///)
      READ (12) HT,HEAD,NF,ND
      N=ND-M
      READ (1,100 ) (KEEP(I),I=1,N )
      NERR=0
      DO 250 L=1,NF
      IX=IZ*2+1
      READ (12) HZ(L),((YR(I,J),YI(I,J),I=1,ND),J=1,ND)
      WRITE (3,120 ) HZ(L)
  120 FORMAT ('1'/T30,'MAIN SYSTEM DISPLACEMENT MOBILITY      REAL,IMAGI
     ANARY       FREQ='F8.2//)
      CALL MOUT2 (YR,ND,ND )
      CALL MOUT2 (YI,ND,ND )
      READ (11) HF(L),((YBR(I,J),YBI(I,J),I=1,M),J=1,M)
      WRITE (3,130) HF(L)
  130 FORMAT ('1'/T30,'  SUBSYSTEM DISPLACEMENT MOBILITY      REAL,IMAGI
     ANARY       FREQ='F7.2//)
      CALL MOUT2 ( YBR,M,M )
      CALL MOUT2 ( YBI,M,M )
      IF(HZ(L)-HF(L)) 140,150,140
  140 WRITE (3,310)
      GO TO 350
  150 IF(PCTR.NE.0.OR.PCTBR.NE.0.OR.PCTI.NE.0.OR.PCTBI.NE.0 ) NERR=1
      IF (NERR.NE.1 ) GO TO 180
       CALL ERRNU (YR,YI,PCTR,PCTBR,PCTI,PCTBI,ND,ND,IX )
      WRITE (3,160 )
  160 FORMAT ('1'/T15,'MAIN SYSTEM DISPLACEMENT MOBILITY WITH ERROR
     AREAL,IMAGINARY    FREQ='F8.2//)
      CALL MOUT2 ( YR,ND,ND )
      CALL MOUT2 ( YI,ND,ND )
      WRITE (3,170 )
  170 FORMAT ('1'/T15,'  SUBSYSTEM DISPLACEMENT MOBILITY WITH ERROR
     AREAL,IMAGINARY    FREQ='F8.2//)
       CALL ERRNU (YBR,YBI,PCTR,PCTBR,PCTI,PCTBI,ND,ND,IX )
      CALL MOUT2 ( YBR,M,M )
      CALL MOUT2 ( YBI,M,M )
C
  180 DO 190 I=1,N
      YAAR(I,1)=YR(KEEP(I),NCOL )
  190 YAAI(I,1)=YI(KEEP(I),NCOL )
      DO 200 I=1,M
      YCR(I,1)=YR(MS(I),NCOL )
  200 YCI(I,1)=YI(MS(I),NCOL )
```

```
      DO 210 I=1,N
      DO 210 J=1,M
      YABR(I,J)=YR(KEEP(I),MS(J))
  210 YABI(I,J)=YI(KEEP(I),MS(J))
      DO 220 I=1,M
      DO 220 J=1,M
      YCCR(I,J)=YR(MS(I),MS(J))
  220 YCCI(I,J)=YI(MS(I),MS(J))
C
  230 K=N+1
      LL=1
      DO 250 I=K,ND
      YAAR(I,1)=0.
      YAAI(I,1)=0.
      DO 240 J=1,M
      YABR(I,J)=-YBR(LL,J)
  240 YABI(I,J)=-YBI(LL,J)
  250 LL=LL+1
      CALL MOUT2 (YAAR,ND,1 )
      CALL MOUT2 (YAAI,ND,1 )
      CALL MOUT2 (YABR,ND,M )
      CALL MOUT2 (YABI,ND,M )
C
C
      CALL MOUT2 (YCR,M,1 )
      CALL MOUT2 (YCI,M,1 )
      CALL MOUT2 (YCCR,M,M )
      CALL MOUT2 (YCCI,M,M )
      CALL MATAS  (YBR,YCCR,M,M,1. )
      CALL MATAS  (YBI,YCCI,M,M,1. )
      CALL CINV (YBR,YBI,M,YCCR,YCCI )
      CALL CMMPY (YABR,YABI,YCCR,YCCI,ND,M,M,YR,YI )
      CALL CMMPY ( YR,YI,YCR,YCI,ND,M,1,YABR,YABI )
      CALL MATAS ( YAAR,YABR,ND,1,-1. )
      CALL MATAS ( YAAI,YABI,ND,1,-1. )
      WRITE (3,260) HZ(L),NCOL,(MS(I),I=1,M )
  260 FORMAT ('1'/T10,'DISPLACEMENT MOBILITY  REAL,IMAGINARY   FREQ='
     A F8.2,' HERTZ'/T20,'FORCING AT MAIN SYSTEM STA='I3/T20,'SUBSYSTEM
     BINTERFACE STA='I3,',',I3,',',I3////)
      WRITE (3,320)
      DO 270 I=1,N
  270 WRITE (3,340 ) YAAR(I,1),YAAI(I,1)
      WRITE (3,330)
      DO 280 I=1,M
  280 WRITE (3,340 ) YAAR(N+I,1),YAAI(N+I,1)
  290 CONTINUE
  300 FORMAT( 4F10.4,I10 )
  310 FORMAT ('1'//'  FORCING FREQUENCIES OF MAIN BODY AND SUBSYSTEM ARE
     A INCOMPATABLE'/'    JOB TERMINATED')
  320 FORMAT ( 20X,'MAIN SYSTEM RESPONSE'/)
  330 FORMAT ( 20X,'INTERFACE   RESPONSE'/)
  340 FORMAT ( 20X,1P2E15.4)
  350 CALL EXIT
      END
```

```
      SUBROUTINE MATAS  ( A,B,N1,N2,S )
C     ADDITION OF MATRICES A(N1,N2) AND B(N1,N2) STORED IN A
      DIMENSION A(20,1),B(20,1)
      DO 100 I=1,N1
      DO 100 J=1,N2
  100 A(I,J)=A(I,J)+S*B(I,J)
      RETURN
      END
```

```
      SUBROUTINE ERRNU (A,B,PCTR,PCTBR,PCTI,PCTBI, NJ,NP,IX)
C
C                                                       A BIAS ERROR,
C             PCTB (RATIO) ON AMPLITUDE, AND A UNIFORM RANDOM ERROR
C             HAVING A +/- MAXIMUM OF PCT (RATIO) ON AMPLITUDE.
C        USES RANDU
C
      DIMENSION A(20,21),B(20,21)
      IF(PCTR) 110,100,110
  100 IF(PCTBR) 110,130,110
  110 DO 120 I=1,NJ
      DO 120 J=1,NP
      CALL RANDU (IX,IY,YFL)
      IX=IY
      E=1.0+2.0*PCTR*(YFL-0.5)+PCTBR
      A(I,J)=A(I,J)*E
      CALL RANDU (IX,IY,YFL)
      IX=IY
      E=1.0+2.0*PCTI *(YFL-0.5)+PCTBI
  120 B(I,J)=B(I,J)*E
  130 RETURN
      END
```

```
      SUBROUTINE MOUT2 (A,M,N)
      REAL A(20,21)
      ID=MINO(N,10)
      WRITE (3,100) (I,I=1,ID)
  100 FORMAT (/T5,10I12)
      WRITE (3,100)
      DO 11C I=1,M
  110 WRITE (3,120) I,(A(I,J),J=1,ID)
  120 FORMAT (I5,5X,1P10E12.4)
      IF (ID-N) 130,170,170
  130 ID=MINO(N,20)
      WRITE (3,100) (I,I=11,ID )
      WRITE (3,100)
      DO 14C I=1,M
  140 WRITE (3,120) I,(A(I,J),J=11,ID )
      IF(ID-N) 150,170,170
  150 WRITE (3,100) (I,I=21,N )
      WRITE (3,100 )
      DO 16C I=1,M
  160 WRITE (3,120 ) I,( A(I,J),J=21,N )
  170 RETURN
      END
```

```
      SUBROUTINE MMPY (A,B,N1,N2,N3,C)
C
C                                C = A * B
C                                A (N1 X N2)    B (N2 X N3)    C (N1 X N3)
C
      REAL A(20,21),B(20,21),C(20,21)
      DO 100 I=1,N1
      DO 100 J=1,N3
      C(I,J)=C.
      DO 100 K=1,N2
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE INVRS (B,N,A)                                          INV
C     A = INVERSE OF B          B UNDISTURBED                           INV
C                                                                       INV
      DIMENSION A(20,21),D(20,21),IROW(21),ICOL(21),B(20,21             INV
      DO 100 I=1,N                                                     1INV
      DO 100 J=1,N                                                     2INV
  100 A(I,J)=B(I,J)                                                    2INV
      M=N+1                                                             INV
      DO 110  I=1,N                                                    1INV
      IROW(I)=I                                                        1INV
  110 ICOL(I)=I                                                        1INV
      DO 260 K=1,N                                                     1INV
      AMAX= A(K,K)                                                     1INV
      DO 130 I=K,N                                                     2INV
      DO 130 J=K,N                                                     3INV
      IF(ABS( A(I,J))-ABS(AMAX))130,120,120                            3INV
  120 AMAX= A(I,J)                                                     3INV
      IC=I                                                             3INV
      JC=J                                                             3INV
  130 CONTINUE                                                         3INV
      KI=ICOL(K)                                                       1INV
      ICOL(K)=ICOL(IC)                                                 1INV
      ICOL(IC)=KI                                                      1INV
      KI=IROW(K)                                                       1INV
      IROW(K)=IROW(JC)                                                 1INV
      IROW(JC)=KI                                                      1INV
      IF(AMAX) 160,140,160                                             1INV
  140 WRITE (3,150)                                                    1INV
  150 FORMAT(' SOLUTION OF EXISTING MATRIX NOT POSSIBLE')              1INV
      GO TO 330                                                        1INV
  160 DO 170 J=1,N                                                     2INV
      E=A(K,J)                                                         2INV
      A(K,J)=A(IC,J)                                                   2INV
  170 A(IC,J)=E                                                        2INV
      DO 180 I=1,N                                                     2INV
      E=A(I,K)                                                         2INV
      A(I,K)=A(I,JC)                                                   2INV
  180 A(I,JC)=E                                                        2INV
      DO 210 I=1,N                                                     2INV
      IF(I-K) 200,190,200                                              2INV
  190 A(I,M)=1.                                                        2INV
      GO TO 210                                                        2INV
  200 A(I,M)=0.                                                        2INV
  210 CONTINUE                                                         2INV
      PVT=A(K,K)                                                       1INV
      DO 220 J=1,M                                                     2INV
  220 A(K,J)=A(K,J)/PVT                                                2INV
      DO 250 I=1,N                                                     2INV
      IF(I-K)230,250,230                                               2INV
  230 AMULT=A(I,K)                                                     2INV
      DO 240 J=1,M                                                     3INV
  240 A(I,J)=A(I,J)-AMULT*A(K,J)                                       3INV
  250 CONTINUE                                                         2INV
      DO 260 I=1,N                                                     2INV
  260  A(I,K)=A(I,M)                                                   2INV
```

```
      DO 290 I=1,N
      DO 270 L=1,N
      IF(IROW(I)-L)270,280,270
  270 CONTINUE
  280 DO 290 J=1,N
  290 C(L,J)=A(I,J)
      DO 320 J=1,N
      DO 300 L=1,N
      IF(ICOL(J)-L) 300,310,300
  300 CONTINUE
  310 DO 320 I=1,N
  320 A(I,L)=C(I,J)
  330 RETURN
      END
```

```
      SUBROUTINE RANDU (IX,IY,YFL)
C                      THIS SUBROUTINE IS FROM  SSP VERS. II
      IY=IX*65539
      IF(IY) 100,110,110
  100 IY=IY+2147483647+1
  110 YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END
```

```
      SUBROUTINE CINV (A,B,N,C,D)
C
C                               C+I*D = INVERSE OF A+I*B       I=SQRT(-1)
C                               B ASSUMED NON SINGULAR
C
      REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)
      CALL INVRS(B,N,C)
      CALL MMPY(C,A,N,N,N,E)
      CALL MMPY(A,E,N,N,N,C)
      DO 100 I=1,N
      DO 100 J=1,N
  100 C(I,J)=C(I,J)+B(I,J)
      CALL INVRS(C,N,D)
      CALL MMPY(E,D,N,N,N,C)
      DO 110 I=1,N
      DO 110 J=1,N
  110 D(I,J)=-D(I,J)
      RETURN
      END
```

```
      SUBROUTINE CMMPY (A,B,C,D,N1,N2,N3,E,F )
C
C        COMPLEX MATRIX MULT
C        E + I*F =(A + I*B)*(C + I*D)       I = SQR(-1)
C        A,B ARE N1 X N2      C,C ARE N2 X N3      E,F ARE N1 X N3
C
      DIMENSION A(20,21),B(20,21),C(20,21),D(20,21),E(20,21),F(20,21)
     A,G(20,21)
      CALL MMPY  (A,C,N1,N2,N3,E )
      CALL MMPY  (B,D,N1,N2,N3,G )
      DO 100 I=1,N1
      DO 100 J=1,N3
  100 E(I,J)=E(I,J)-G(I,J)
      CALL MMPY  (A,D,N1,N2,N3,F )
      CALL MMPY ( B,C,N1,N2,N3,G )
      DO 110 I=1,N1
      DO 110 J=1,N3
  110 F(I,J)=F(I,J)+G(I,J)
      RETURN
      END
```